

# Superfluid phases of triplet pairing and rapid cooling of the neutron star in Cassiopeia A

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## Abstract

In a simple model it is demonstrated that the neutron star surface temperature evolution is sensitive to the phase state of the triplet superfluid condensate. A multicomponent triplet pairing of superfluid neutrons in the core of a neutron star with participation of several magnetic quantum numbers leads to neutrino energy losses exceeding the losses from the unicomponent pairing. A phase transition of the neutron condensate into the multicomponent state triggers more rapid cooling of superfluid core in neutron stars. This makes it possible to simulate an anomalously rapid cooling of neutron stars within the minimal cooling paradigm without employing any exotic scenarios suggested earlier for rapid cooling of isolated neutron star in Cassiopeia A.

*Keywords:* Neutron star, Superfluidity, Neutrino radiation

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## 1. Introduction

Studying thermal evolution of isolated neutron stars in X-rays is of a great importance for better understanding the evolution of such objects and provides a possibility to investigate their composition and structure (see e.g., [1, 2, 3]). The thermal X-ray radiation from the neutron star (NS) at the center of the Cassiopeia A (Cas A) supernova remnant<sup>1</sup> attracts much attention nowadays. A few years ago Heinke & Ho [7, 8] have analyzed Chandra

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<sup>1</sup>The supernova remnant in Cassiopeia A contains a young ( $\approx 330$  yr old [4]) neutron star which was discovered by Chandra satellite [5, 6] in 1999.

observation data during 10 years and reported an anomalous steady decline of the surface temperature,  $T_s$ . The authors have interpreted this data as a direct observation of Cas A NS cooling, the phenomenon which has never been observed before for any isolated NS.

We shall discuss later the current state of these observations, at the moment we note that although the real cooling rate is under debate one can not exclude that the Cas A NS cooling is extraordinarily fast. Even a 1% decline of the cooling curve in 10 years would signal very fast cooling. Such a rapid drop in surface temperature (if it occurs) is in conflict with standard cooling scenarios based on the efficient modified Urca process. If the NS in Cas A underwent standard cooling (through neutrino emission from the core due to the modified Urca process) its surface temperature decline in 10 years would be 0.2% – 0.3% [9, 10].

The rapid decline but relatively high surface temperature (about  $2.12 \times 10^6$  K) require a dramatic change in the neutrino emission properties of the NS. Some exotic scenarios of cooling have been suggested that employ nonstandard assumptions on NS physics and evolution, involving softened pion modes [11], quarks [12, 13], axions [14] or cooling after an r-mode heating process [15]. An existence of softened pions or quarks in the NS core depends mostly on the matter density but not on the temperature. If this rapid cooling was constant from the birth of the NS, the current temperature would have to be much smaller than is currently measured.

It is reasonable to suggest [16, 17] that the cooling was initially slow but greatly accelerated later. In this case the rapid temperature decline could be naturally explainable in a frame of the minimal cooling paradigm [1, 2] that assumes that rapid cooling of the neutron star is triggered by neutron superfluidity in the core. This scenario implies that neutrons have recently become superfluid (in  $^3P_2$  triplet-state) in the NS core, triggering a huge neutrino flux from pair breaking and formation (PBF) processes that accelerates the cooling [16, 17], while protons were already in a superconducting  $^1S_0$  singlet-state with a larger critical temperature. Although the above mechanism is consistent with the commonly accepted cooling paradigm, the theoretical simulation has shown [17, 18], that the PBF processes in the neutron triplet condensate are not enough effective to explain the rapid temperature decline. This has stimulated the present work.

It is commonly believed [19, 20, 21, 22] that the pair condensation in the superdense neutron matter occurs into the  $^3P_2$  state (with a small admixture of  $^3F_2$ ) with a preferred magnetic quantum number  $m_j = 0$ . This model has

been conventionally used for estimates of the PBF neutrino energy losses in the minimal cooling scenarios.

Let us remind that, in the case of  ${}^3\text{P}_2$  ( $m_j = 0$ ) pairing, the PBF  $\bar{\nu}\nu$  emissivity is evaluated as [23] (we use natural units,  $\hbar = c = k_B = 1$ ):

$$Q(m_j = 0) \simeq \frac{2}{5\pi^5} G_F^2 C_A^2 p_F M^* T^7 F(T/T_c) , \quad (1)$$

where  $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi coupling constant,  $C_A$  is the axial-vector coupling constant of neutrons,  $p_F$  is the Fermi momentum of neutrons,  $M^* \equiv p_F/V_F$  is the neutron effective mass; the function  $F$  is given by

$$F(T/T_c) = \int \frac{d\mathbf{n}}{4\pi} \frac{\Delta_{\mathbf{n}}^2}{T^2} \int_0^\infty dx \frac{z^4}{(\exp z + 1)^2}, \quad (2)$$

where  $z = \sqrt{x^2 + \Delta_{\mathbf{n}}^2/T^2}$ , and the superfluid energy gap,

$$\Delta_{\mathbf{n}}(\theta, T) = \sqrt{\frac{1}{2} (1 + 3 \cos^2 \theta)} \Delta(T), \quad (3)$$

is anisotropic. It depends on polar angle  $\theta$  of the quasiparticle momentum and temperature<sup>2</sup>.

In the present letter I argue that the enlarged neutrino energy losses can be explained in terms of the conventional minimal cooling paradigm assuming that the enhanced neutrino radiation can be a natural consequence of the phase transition of the  ${}^3\text{P}_2$  condensate into a multicomponent state.

Modern calculations [25, 26] have shown that, besides the one-component state with  $m_j = 0$ , there are also multicomponent  ${}^3\text{P}_2$  states involving several magnetic quantum numbers  $m_j = 0, \pm 1, \pm 2$  that compete in energy and represent various phases of the condensate in equilibrium<sup>3</sup>. The general form

<sup>2</sup>Notice that our definition of the gap amplitude differs from the gap definition used, in Ref. [24] by the factor of  $\sqrt{2}$ .

<sup>3</sup>Do not confuse with "angulons" which represent Goldstone bosons associated with broken rotational symmetry in a  ${}^3\text{P}_2$  ( $m_j = 0$ ) condensed neutron superfluid [27]. These collective excitations represent small angular oscillations of the condensate. The complete set of the oscillation modes of the  ${}^3\text{P}_2$  ( $m_j = 0$ ) condensate in the superfluid neutron liquid is analyzed in [28]. Neutrino emission due to decay of these collective oscillations produces a negligibly small contribution into the NS cooling [29].

of a unitary  ${}^3\text{P}_2$  state includes  $m_j = 0, \pm 1, \pm 2$ , and the superfluid energy gap can be defined by the relation [30]

$$D^2(\mathbf{n}, \tau) = \bar{\mathbf{b}}^2(\mathbf{n}) \Delta^2(\tau), \quad (4)$$

where  $\tau \equiv T/T_c$  is the relative temperature; the (temperature dependent) gap amplitude is of the form

$$\Delta^2 = \Delta_0^2 + 2\Delta_1^2 + 2\Delta_2^2, \quad (5)$$

and  $\bar{\mathbf{b}}(\mathbf{n})$  is a real vector normalized by the condition

$$\langle \bar{b}^2(\mathbf{n}) \rangle \equiv (4\pi)^{-1} \int \bar{b}^2(\mathbf{n}) d\mathbf{n} = 1. \quad (6)$$

Its angular dependence is represented by the unit vector  $\mathbf{n} = \mathbf{p}/p$  which defines the polar angles  $(\theta, \varphi)$  on the Fermi surface:

$$\mathbf{n} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \equiv (n_1, n_2, n_3). \quad (7)$$

The properly normalized vector  $\bar{\mathbf{b}}$  can be written by utilizing notation adopted in Refs. [25, 26], where  $\lambda_1 \equiv \sqrt{6}\Delta_1/\Delta_0$  and  $\lambda_2 \equiv \sqrt{6}\Delta_2/\Delta_0$ :

$$\bar{\mathbf{b}} = \sqrt{\frac{1}{2} \frac{\Delta_0}{\Delta}} \begin{pmatrix} -n_1 + n_1\lambda_2 - n_3\lambda_1, & -n_2 - n_2\lambda_2, & 2n_3 - n_1\lambda_1 \end{pmatrix}. \quad (8)$$

According to modern theories, there are several multicomponent states that compete in energy depending on the temperature. Accordingly the phase transitions can occur between these states when the temperature goes down. The possible phase states of the  ${}^3\text{PF}_2$  condensate are cataloged in Ref. [25].

In Table 1 we have collected the nodeless states which are especially interesting. Immediately below the critical temperature, the superfluid condensate can appear in either the one-component phase  $O_0$ , corresponding to  $m_j = 0$ , or in one of the two two-component phases,  $O_{\pm 3}$ . These lowest-energy states are nearly degenerate. The higher nearly degenerate group is composed of the phases  $O_1$  and  $O_2$ .

The energy split between the two groups shrinks along with the temperature decrease [25] and can result in a phase transition at some temperature<sup>4</sup>

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<sup>4</sup>Authors predict the transition temperature  $T \simeq 0.7T_c$  at  $p_F \simeq 2.1 \text{ fm}^{-1}$ .

Table 1: Various phases of the  ${}^3\text{P}_2$  condensate and their relative neutrino emissivity  $Z$

phase	$\Delta_0/\Delta$	$\lambda_1$	$\lambda_2$	$Z$
$O_0$	1	0	0	1
$O_{\pm 3}$	$\frac{1}{2}$	0	$\pm 3$	3.25
$O_1$	$\frac{5}{\sqrt{14}\sqrt{17-3\sqrt{21}}}$	$\frac{3}{5}\sqrt{2(17-3\sqrt{21})}$	$\frac{3}{5}(\sqrt{21}-4)$	2.3528
$O_2$	$\frac{5}{\sqrt{14}\sqrt{17+3\sqrt{21}}}$	$\frac{3}{5}\sqrt{2(17+3\sqrt{21})}$	$-\frac{3}{5}(\sqrt{21}+4)$	3.8258

$T < T_c$ , depending on the matter density. The small difference in the gap amplitudes,  $\sim 2\%$ , inherent for various phases of the condensate, is crucial for the phase transitions, but this small inequality can be disregarded in evaluation of the neutrino energy losses.

## 2. Neutrino emission from a multicomponent phase

The neutrino emissivities of the multicomponent phase states have been analyzed in Ref. [30] in the approximation of averaged gap. The calculation technique, developed in that work, allows us to derive a more accurate expression taking into account the gap anisotropy. To this end we have to use Eq (68) of Ref. [30] and the polarization tensor, as given just below Eq (65). Starting from these expressions we consider the case of  $\omega^2 > 2\bar{\mathbf{b}}^2\Delta^2$  which is fulfilled for the PBF processes. Then after performing integrations over  $d^3q$  one can obtain the neutrino energy losses per unit volume and time in the  $\Lambda$  state (we abbreviate the set of numbers  $\Delta_0/\Delta, \lambda_1, \lambda_2$  as  $\Lambda$ ).

$$Q_\Lambda = \frac{2}{5\pi^5} C_A^2 G_F^2 p_F M^* T^7 F_\Lambda(\tau) , \quad (9)$$

where

$$F_\Lambda(\tau) = \left(4 - 3\frac{\Delta_0^2}{\Delta^2}\right) y^2 \int \frac{d\mathbf{n}}{4\pi} \bar{b}^2(\mathbf{n}) \int_0^\infty dx \frac{z^4}{(1 + \exp z)^2} \quad (10)$$

with  $z = \sqrt{x^2 + \bar{b}^2(\mathbf{n})} y$ ,  $y(\tau) = \Delta(T)/T$ , and the function  $\bar{b}^2(\mathbf{n})$  is given by

$$\begin{aligned} \bar{b}^2(\mathbf{n}) = & \frac{1}{4} \frac{\Delta_0^2}{\Delta^2} [2 + \lambda_1^2 + 2\lambda_2^2 + (6 + \lambda_1^2 - 2\lambda_2^2) \cos^2 \theta \\ & - 2\lambda_1(1 + \lambda_2) \sin 2\theta \cos \varphi + (\lambda_1^2 - 4\lambda_2) \sin^2 \theta \cos 2\varphi] \end{aligned} \quad (11)$$

At  $\lambda_1 = \lambda_2 = 0$  and  $\Delta = \Delta_0$  the expression (9) recovers Eq. (1).

For numerical evaluation of the neutrino losses, as given in Eq. (9), it is necessary to know the function  $y(\tau) = \Delta(T)/T$ , which in general is to be found with the aid of gap equations. However, as mentioned above, the difference in the gap amplitudes for various phases can be neglected in evaluation of the neutrino energy losses. This substantially simplifies the problem because for the case  $m_j = 0$  the function is well investigated<sup>5</sup>.

### 3. Modeling of the cooling process

To get an idea of how the phase state of the superfluid condensate can influence the NS surface temperature let us consider a simple model of cooling of the superfluid neutron core enclosed in a thin envelope.

We assume that the bulk matter consists mostly of  ${}^3\text{P}_2$  superfluid neutrons. The neutrino emission due to  ${}^1\text{S}_0$  proton pairing is strongly suppressed in the non-relativistic system [31, 32], but the energy gap arising in the quasi-particle spectrum below the condensation temperature suppresses the most mechanisms of neutrino emission which are efficient in the normal (nonsuperfluid) nucleon matter ( $\nu\bar{\nu}$  bremsstrahlung, modified Urca processes etc.) [33]. As was found in Ref. [17, 16] this scenario puts stringent constraints on the temperature for the onset of neutron superfluidity in the Cas A NS. Namely, the transition temperature dependence on the density should have a wide peak with maximum  $T_c(\rho) \approx (5 - 8) \times 10^8$  K.

In the temperature range which we are interested in, the thermal luminosity of the surface is negligible in comparison to the neutrino luminosity of PBF processes in the NS core. In this case the equation of global thermal balance [34] reduces to

$$C(\tilde{T}) \frac{d\tilde{T}}{dt} = -L(\tilde{T}). \quad (12)$$

Here  $L(\tilde{T})$  is the total PBF luminosity of the star (redshifted to a distant observer), while  $C(\tilde{T})$  is the stellar heat capacity. These quantities are given

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<sup>5</sup>We use the simple fit  $\sqrt{2}v_B(\tau)$  suggested in Ref. [24].

by (see details in Ref. [35]):

$$L(\tilde{T}) = \int dV Q_{\Lambda}(T, \rho) \exp(2\Phi(r)), \quad (13)$$

$$C(\tilde{T}) = \int dV C_V(T, \rho), \quad (14)$$

where  $C_V(T, \rho)$  is the specific heat capacity,

$$dV = 4\pi r^2 \left(1 - \frac{2Gm(r)}{r}\right)^{-1/2} dr,$$

where  $G$  stands for gravitation constant,  $m(r)$  is the gravitational mass enclosed within radius  $r$ , and  $\Phi(r)$  is the metric function that determines gravitational redshift. A thermally relaxed star has an isothermal interior which extends from the center to the heat blanketing envelope. Following [34] we have assumed that the isothermal region is restricted by the condition  $\rho > \rho(r_b) = 10^{10} \text{ g cm}^{-3}$ . Taking into account the effects of General Relativity (e.g., [36]), isothermality at  $r < r_b$  means spatially constant redshifted internal temperature  $\tilde{T}(t)$ , while the local internal temperature

$$T(r, t) = \tilde{T}(t) \exp(-\Phi(r)), \quad (15)$$

depends on radial coordinate  $r$ . Generally, the redshift factor has to be calculated using the Tolman-Oppenheimer-Volkoff equation. In vacuum, outside the star and at the stellar surface this factor is of the form

$$\exp \Phi(r) = \left(1 - \frac{2Gm(r)}{r}\right)^{1/2}. \quad (16)$$

For simplicity we shall use this expression in the crust of the star, as a model.

The main temperature gradient is formed in the thermally insulating outer envelope at  $r > r_b$ . Since the envelope is thin one can set  $r_b \simeq R$  and  $m(r_b) \simeq M$ , where  $R$  and  $M$  are the radius and mass of the NS, respectively. Then the temperature  $T_b = T(r_b)$  at the bottom of the thermally insulating envelope of the star can be written as

$$T_b = \left(1 - \frac{R_g}{R}\right)^{-1/2} \tilde{T}, \quad (17)$$

where

$$R_g \equiv 2GM \simeq 2.953 \frac{M}{M_\odot} \text{ km} \quad (18)$$

is the Schwarzschild radius.

One can convert the internal  $T_b$  to the observed effective surface temperature  $T_s$  using the simple analytical relationship found by Gundmundsson, Pethick & Epstein [38]:

$$T_s/10^6 \text{ K} \simeq 0.87 g_{s14}^{1/4} (T_b/10^8 \text{ K})^{0.55}. \quad (19)$$

Here  $g_{s14} = g_s/10^{14} \text{ cm s}^{-2}$  where

$$g_s = \frac{GM}{R^2 \sqrt{1 - R_g/R}} \simeq \frac{1.328 \times 10^{14}}{\sqrt{1 - R_g/R}} \frac{M/M_\odot}{R_6^2} \text{ cm s}^{-2}, \quad (20)$$

with  $R_6 \equiv R/(10^6 \text{ cm})$ , is the acceleration of gravity as measured at the surface.

Given the strong dependence of the PBF processes on the temperature  $T$  and density  $\rho$ , the overall effect of emission of neutrino pairs can only be assessed by complete calculations of the neutron star cooling which are beyond the scope of this paper. We do not aim to carry out exact calculations. Our goal is to demonstrate that the NS cooling rate substantially depends on the phase state of the  ${}^3\text{P}_2$  condensate of superfluid neutrons. A rough estimate can be made in a simplified model, where both the superfluid transition temperature,  $T_c$ , and the real temperature,  $T = T_{\text{core}}$ , are constant over the core.

In the temperature range of our interest, the specific heat is governed by the neutron component (the contribution of electrons and strongly superfluid protons is negligibly small) and can be described as

$$C \simeq \frac{1}{3} T_{\text{core}} R_B(T_{\text{core}}/T_c) \int dV p_F M^*, \quad (21)$$

where  $R_B(\tau)$  is the superfluid reduction factor, as given in Eq. (18) of Ref. [33].

Making use of Eq. (9) we obtain the PBF luminosity in the form

$$L = \frac{2}{5\pi^5} G_F^2 C_A^2 T_{\text{core}}^7 F_\Lambda(T_{\text{core}}/T_c) \int dV p_F M^* e^{2\Phi(r)}, \quad (22)$$



where  $F_\Lambda(\tau)$  is given by Eq. (10).

Insertion of Eqs. (15), (21) and (22) into Eq. (12) allows us to obtain the following equation for the non-redshifted temperature  $T(r_{\text{core}}, t) \equiv T_{\text{core}}(t)$  at the edge of the core, at  $r = r_{\text{core}}$ :

$$\frac{dT_{\text{core}}}{dt} = -\frac{3\alpha}{R_B (T_{\text{core}}/T_c)} \frac{2}{5\pi^5} G_F^2 C_A^2 T_{\text{core}}^6 F_\Lambda (T_{\text{core}}/T_c). \quad (23)$$

Here the constant  $\alpha \equiv \alpha(r_{\text{core}})$  is defined as

$$\alpha \equiv \frac{\int dV p_F M^* e^{2\Phi(r)}}{\exp \Phi(r_{\text{core}}) \int dV p_F M^*}, \quad (24)$$

where the integration is over the core volume,  $r \leq r_{\text{core}}$ .

In Eq. (23)  $T_{\text{core}}$  is the real temperature in the core, particularly, at the crust-core interface which corresponds to the density of about  $1.5 \times 10^{14}$  g/cm<sup>3</sup> at  $r = r_{\text{core}}$ . One can convert it to the redshifted internal temperature  $\tilde{T}(t)$  as

$$\tilde{T} = \left(1 - \frac{2Gm(r_{\text{core}})}{r_{\text{core}}}\right)^{1/2} T_{\text{core}} \simeq \left(1 - \frac{R_g}{r_{\text{core}}}\right)^{1/2} T_{\text{core}}. \quad (25)$$

When obtaining the second equality we have neglected the mass of the crust which is small ( $\sim 1\%$ ) in comparison with the mass of the core [37]. This allows us to set  $m(r_{\text{core}}) \simeq M$ .

From Eqs. (17) and (25) one can find the temperature at the bottom of the thermally insulating envelope

$$T_b = \left(1 - \frac{R_g}{R}\right)^{-1/2} \left(1 - \frac{R_g}{r_{\text{core}}}\right)^{1/2} T_{\text{core}}. \quad (26)$$

Insertion of this expression into Eq. (19) allows one to find the observed (non-redshifted) surface temperature  $T_s$ :

$$T_s/10^6 \text{K} \simeq 0.87 g_{s14}^{1/4} \left(\frac{1 - R_g/r_{\text{core}}}{1 - R_g/R}\right)^{\frac{0.55}{2}} (T_{\text{core}}/10^8 \text{K})^{0.55}. \quad (27)$$

Assuming that the crust thickness is about  $0.1R$  [37] one can set  $r_{\text{core}} \simeq 0.9R$ .

We adopt  $R = 10.3$  km and  $M = 1.65M_\odot$ . In this case

$$0.87 g_{s14}^{1/4} \left(\frac{1 - R_g/r_{\text{core}}}{1 - R_g/R}\right)^{\frac{0.55}{2}} \simeq 1.098, \quad (28)$$

which yields

$$T_s/10^6\text{K} \simeq 1.098(T_{\text{core}}/10^8\text{K})^{0.55}. \quad (29)$$

Thus our simulation of the NS cooling is reduced to numerical solving of Eqs. (23) and (29).

#### 4. Simulation results

In Fig. 1 we demonstrate the cooling curves of the superfluid neutron star with a constant  $T_c$  over the core. The curves obtained for the superfluid phases listed in Table 1 and are labeled respectively.

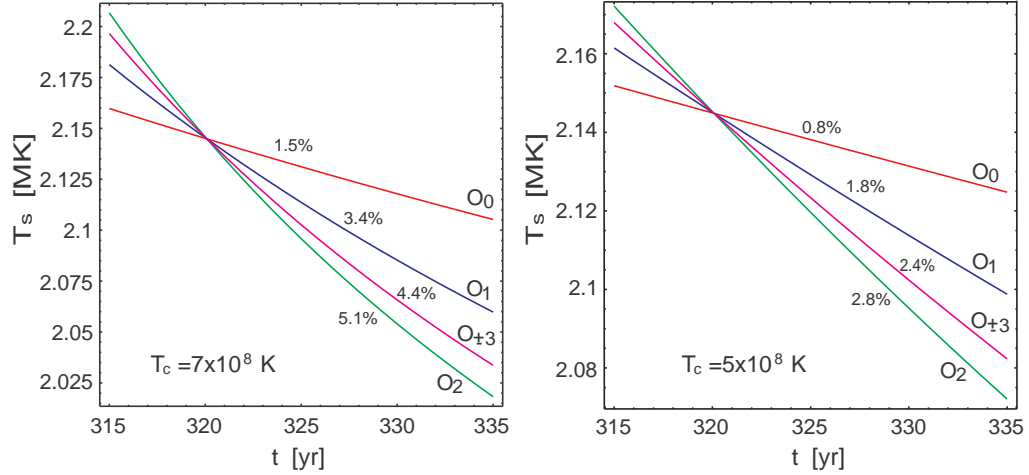


Figure 1: (Color on line) *Left*: Cooling curves for Cas A NS which has a superfluid neutron core and a low-mass heat blanketing envelope.  $T_c = 7 \times 10^8$  K is taken constant over the core. Four curves correspond to different phases of triplet pairing.  $O_0$  is the cooling curve of the one-component phase  $m_j = 0$ . The remaining curves correspond to the  $O_1$ ,  $O_2$ , and  $O_{\pm 3}$  phases. Calculated temperature declines over 10 years are given near the curves (in percent). *Right*: Same but with  $T_c = 5 \times 10^8$  K.

The case  $O_0$  corresponds to the one-component state of the neutron superfluid with  $m_j = 0$ . The remaining three curves correspond to the phases  $O_1$ ,  $O_2$  and  $O_{\pm 3}$ . Two panels of Figure 1 demonstrate the corresponding simulated cooling curves for the cases of  $T_c = 7 \times 10^8$  K and  $T_c = 5 \times 10^8$  K. We show the cooling curves over a period of about 25 years including 10 years of observations. Note that we show the non-redshifted effective surface

temperature. Calculated temperature declines over 10 years are given near the curves (in percent). As it is seen from these curves a satisfactory agreement with observable temperature declines can be easily obtained by a proper choice of the phase state of  ${}^3\text{P}_2$  condensate and adjusting the parameters of superfluidity.

Certainly the approximation of the constant superfluid transition temperature over the neutron star core is too crude, and simulations with realistic  $T_c(\rho)$  profile can be more persuasive. Such a numerical simulation is beyond the scope of this work. Although it is necessary to note that similar simulations were done in [18], where five phenomenological  $T_c(\rho)$  profiles over the NS core were considered, but the free parameter was used for artificial increase of the PBF neutrino emissivity from the  ${}^3\text{P}_2(m_j = 0)$  pairing. These more realistic calculations are in agreement with our qualitative estimates. Our primary goal is to clarify the possible origin for the increased neutrino losses.

One can make a simple estimate of the relative efficiency of PBF processes for various phases of the superfluid neutron matter. To this end we can evaluate Eq. (9) in the approximation of averaged gap that reduces to the replacement  $\bar{b}^2 \rightarrow \langle \bar{b}^2 \rangle = 1$ . We then recover the result obtained in Eq. (74) of Ref. [30]):

$$\bar{Q}_\Lambda \simeq Z(\Lambda) \bar{Q}(m_j = 0) , \quad (30)$$

where  $\bar{Q}(m_j = 0)$  is given by Eq. (1) but with a replacement  $\Delta_{\mathbf{n}}^2 \rightarrow \Delta^2$ , and

$$Z(\Lambda) = \left( 4 - 3 \frac{\Delta_0^2}{\Delta^2} \right) , \quad (31)$$

These factors representing the relative efficiency of PBF processes for various phases of the  ${}^3\text{P}_2$  superfluid neutron matter are shown in Table 1.

## 5. Discussion and conclusion

Our simple analytic expression (9) for the PBF neutrino emissivity from the multicomponent phases of the  ${}^3\text{P}_2$  superfluid neutron liquid shows that the PBF neutrino losses from the multicomponent condensate can be a few times larger than the corresponding neutrino losses from the one-component condensate with  $m_j = 0$ .

We have employed Eq. (9) for a simple cooling model of a superfluid neutron core enclosed in a thin envelope assuming that the superfluid transition temperature  $T_c$  is constant over the core. In this simple model we have

demonstrated that the NS surface temperature is sensitive to the phase state of the superfluid condensate of neutrons, and this allows one to qualitatively explain the anomalously rapid cooling of the Cas A NS (if it occurs). In other words, we have demonstrated the principal possibility of simulations of rapid cooling in frame of the minimal cooling paradigm without any artificial change of the PBF neutrino emissivity from the  ${}^3\text{P}_2(m_j = 0)$  pairing, as was suggested in Refs. [17, 18]. In a realistic case the superfluid transition temperature  $T_c$  as well as the phase state of the condensate are dependent on the matter density and therefore the phase state of the superfluid liquid can vary along with the distance from the core center. However, the qualitative effects will not be modified by the inclusion of more realistic physics. All the effects discussed above make it possible to explain an anomalously rapid cooling of NSs in many details.

The involving relevance of the multicomponent condensation of neutrons into simulation of the Cas A NS cooling depends on its actual cooling rate which is controversial at the moment. Heinke & Ho [7, 8] have analyzed the archival data from the Chandra X-ray Observatory ACIS-S detector in Graded mode between 2000 and 2009 and reported a steady decline of the surface temperature,  $T_s$ , by about 4%. New observational work on Cas A has shown, however, that the above mentioned rapid cooling of the Cas A NS is not so evident due to systematic uncertainties inherent in the observations and associated with calibration problems of Chandra detectors [18, 39].

Elshamouty et al. [18] compared the results from all the Chandra detectors and found the weighted mean of the temperature decline rate of  $2.9 \pm 0.5_{\text{stat}} \pm 1_{\text{sys}}\%$  over 10 years of observations using the data of all detectors, and a weaker decline of  $1.4 \pm 0.6_{\text{stat}} \pm 1_{\text{sys}}\%$  excluding the data from the ASIS-S detector in the graded mode which suffers from the grade migration.

In contrast, Posselt et al. [39] do not confirm the existence of statistically significant temperature decline and attribute the observed effect to the degradation of the Chandra ASIS-S detector in soft channels. The authors state that the previously reported rapid cooling of the Cas A NS is likely a systematic artifact, and they cannot exclude the standard slow cooling for this NS. Their results (2006-2012) are consistent with no temperature decline at all, or a smaller temperature decline than that reported before although the involved uncertainties are too large to firmly exclude the previously reported fast cooling.

Further observations are necessary to assess the rate of temperature drop with higher accuracy. Let us notice, however, that the discussed problem of

the multicomponent condensation of neutrons can be of interest not only to the Cas A NS cooling but can be relevant also for other superfluid NSs.

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